Effective Interactions of Relativistic Composite Particles in Unified Nonlinear Spinor-Field Models. III*

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Unified nonlinear spinor field models are selfregularizing quantum field theories in which all observable (elementary and non-elementary) particles are assumed to be bound states of fermionic preon fields. Due to their large masses the preons themselves are confined. In preceding papers a functional energy representation, the statistical interpretation and the dynamic equations were derived. In this paper the dynamics of composite particles is discussed. The composite particles are defined to be eigensolutions of the diagonal part of the energy representation. Corresponding calculations are in preparation, but in the present paper a suitable composite particle spectrum is assumed. It consists of preon-antipreon boson states and three-preon-fermion states with corresponding antifermions and contains bound states as well as preon scattering states. The state functional is expanded in terms of these composite particle states with inclusion of preon scattering states. The transformation of the functional energy representation of the spinor field into composite particle functional operators produces a hierarchy of effective interactions at the composite particle level, the leading terms of which are identical with the functional energy representation of a phenomenological boson-fermion coupling theory. This representation is valid as long as the processes are assumed to be below the energetic threshold for preon production or preon break-up reactions, respectively. From this it can be concluded that below the threshold the effective interactions of composite particles in a unified spinor field model lead to phenomenological coupling theories which depend in their properties on the bound state spectrum of the self-regularizing spinor theory.

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8. Evaluation of Cluster Interactions

The cluster representation (5.1) of the operator $\mathcal{H}$ of (1.15) contains two-fermion bound states as well as all scattering states of elementary fermions. This can be seen from the formulae for the cluster representation of the various suboperators given in Section 5. Since the scattering states partly exhibit a negative norm this entails that neither the original representation (1.15) nor the complete cluster representation (5.1) of (1.11) admit a physical interpretation. For a cluster representation there is, however, a chance that a physically meaningful interpretation can be achieved which is not obvious in the original representation: As the bound states of elementary fermions belong to the positive definite part of the spectrum it has to be demonstrated that only these states play the essential role and that all other states can be neglected. This finally leads to a correct statistical interpretation. In order to realize this, two problems have to be solved:

i) the removal of all elementary fermion scattering states from the cluster representation (5.1) must be justified;
ii) the bound state cluster interactions themselves have to be evaluated to give a physically meaningful theory.

While the first problem is a fundamental problem, the second one is concerned only with the elaboration of an already compatible theory. Nevertheless, it is of basic interest, too, as a cluster theory with no relation to phenomenological theories is practically useless.

Both problems are closely related. It is, however, convenient to treat the second problem first.

We decompose $\mathcal{H}$ of (5.1) into two terms

$$\mathcal{H}\left[b, l, \bar{l}, \frac{\delta}{\delta b}, \frac{\delta}{\delta l}, \frac{\delta}{\delta \bar{l}}\right] = \mathcal{H}^{\mathrm{I}}[\ldots] + \mathcal{H}^{\mathrm{II}}[\ldots], \qquad (8.1)$$

where $\mathcal{H}^{\mathrm{I}}$ contains only those cluster operators which belong to bound states, while $\mathcal{H}^{\mathrm{II}}$ contains all remaining cluster operators, i.e. bound state and scattering state cluster operators together as well as only scattering state cluster operators. It follows directly from an inspection of the various terms

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(5.7)−(5.23) that such a decomposition is possible. In the following we will treat $\mathscr{H}^{\mathrm{I}}$ extensively and afterwards sketch the inclusion of $\mathscr{H}^{\mathrm{II}}$, which eventually leads to its exclusion.

For the evaluation of the operator expressions (5.7)−(5.23) we have to calculate integrals which contain the propagator functions $F^r(\boldsymbol{r}, 0)$, $r = 1, 2$. Hence we need a manageable approximation to work successfully with it. The general expression for $F(\boldsymbol{r}, 0)$ reads

$$F(\boldsymbol{r}, 0) = i\gamma \cdot \boldsymbol{r}\left[\frac{1}{4\pi}\,\delta'(r^2) + \frac{m\,i}{4\pi^3}\,r^{-3}\,K_1(m\,r) + \frac{m^2}{4\pi^2}\,r^{-2}\,K_1'(m\,r)\right] + \frac{m}{4\pi}\,\delta(r^2) + \frac{m^2}{4\pi^2}\,r^{-1}\,K_1(m\,r)\,. \tag{8.2}$$

If F is integrated over $\boldsymbol{r}$, the δ'- and δ-distributions drop out and the remaining integral takes a value which is independent of m. This can be verified by the substitution $\boldsymbol{z} = m\boldsymbol{r}$. If F is integrated over $\boldsymbol{r}$ in combination with functions which are normalizable and finite and steady differentiable at the origin, the δ'- and δ-distributions drop out in this case, too. As according to our assumptions these conditions are satisfied by the bound state cluster wave functions, we can omit these terms. Furthermore, as the bound state wave functions are isotropic we assume that the direction dependent parts of (8.2) cancel out in such integrals. Then under the integral, F can be approximated by

$$F(\boldsymbol{r}, 0) \approx \frac{m^2}{4\pi^2}\,r^{-1}\,K_1(m r) \approx \frac{m\,i\,e^{-mr}}{(2\pi)^2\,r^2}\left(\frac{\pi\,m\,r}{2}\right)^{1/2} \tag{8.3}$$

and in this approximation we obtain

$$\int F(\boldsymbol{r}, 0)\,\mathrm{d}^3 r = \Gamma(3/2)\,. \tag{8.4}$$

For very large m (8.3) is a very concentrated function about the origin with a singularity in it. As simultaneously (8.4) holds for all m and independent of m we can consider the set (8.3) as a sequence of test functions converging in the limit of infinite m towards a δ-distribution. We thus make the following assumption:

Assumption 4: In the isotropic approximation the propagator $F(\boldsymbol{r}, 0)$ is approximately given by

$$F(\boldsymbol{r}, 0) \approx \delta(\boldsymbol{r})\;\Gamma(3/2) \tag{8.5}$$

for sufficient large m.

Concerning the sum $(F^1 + F^2)$ which also appears in the integrals it can easily be verified that in the approximation (8.5) this sum yields

$$F^1(\boldsymbol{r}, 0) + F^2(\boldsymbol{r}, 0) \approx \eta^{-1}\,\delta(\boldsymbol{r})\;\Gamma(3/2) \tag{8.6}$$

in the limit of large m_1 and m_2. In deriving (8.6) it has to be observed that the ghost field φ_2 produces a minus sign of the corresponding propagator F^2 compared with the regular propagator F_1. From this it follows furthermore that $(F^1 - F^2)$ which also occurs in the integrals is given by

$$F^1(\boldsymbol{r}, 0) - F^2(\boldsymbol{r}, 0) \approx 2\,\delta(\boldsymbol{r})\;\Gamma(3/2)\,. \tag{8.7}$$

We now turn to the operator expressions. We will explicitly treat only the most important terms. The calculation of all other terms runs along similar lines and will be omitted here.

We first consider the first term of (5.15) (denoted with index a)

$$\begin{aligned}(\mathscr{H}_{bl}^{1})_{\mathrm{a}}^{\mathrm{I}} := &\left(\sum_{nqq'} V_{\alpha\beta\gamma\delta}\,C_n^{\delta\gamma}\,3\,C_q^{\beta u v}\,R_{\alpha u v}^{q'}\,l_{q'}\,\frac{\delta}{\delta l_q}\,\frac{\delta}{\delta b_n}\right)^{\mathrm{I}} \equiv \sum_{\substack{z r r' r''\\ u v}}\,\sum_{s t t'}\,(-1)^z \int \hat V_{\alpha\beta\gamma\delta}\,\delta(\boldsymbol{r}-\boldsymbol{x})\,\delta(\boldsymbol{r}-\boldsymbol{y})\,\delta(\boldsymbol{r}-\boldsymbol{z})\\ &\cdot e^{i\boldsymbol{k}(\boldsymbol{z}+\boldsymbol{y})1/2}\,\varphi_{0,s}(\underset{\delta}{\overset{r''}{\boldsymbol{z}}} - \underset{\gamma}{\overset{r'}{\boldsymbol{y}}})\cdot 3\,e^{i\boldsymbol{q}(\boldsymbol{x}+\boldsymbol{y}'+\boldsymbol{z}')1/3}\,\varphi_{0,t}(\underset{\beta}{\overset{r}{\boldsymbol{y}'}} - \boldsymbol{z}', \underset{\varkappa}{\overset{u}{\boldsymbol{x}}} - \tfrac{1}{2}\,(\underset{\varkappa'}{\overset{v}{\boldsymbol{y}'}} + \boldsymbol{z}'))\;\mathrm{e}^{-i\boldsymbol{q}'(\boldsymbol{r}+\boldsymbol{y}'+\boldsymbol{z}')1/3}\,\bar\varphi_{0,t'}(\underset{\alpha}{\overset{z}{\boldsymbol{y}'}} - \boldsymbol{z}', \underset{\varkappa}{\overset{u}{\boldsymbol{r}}} - \tfrac{1}{2}\,(\underset{\varkappa'}{\overset{v}{\boldsymbol{y}'}} + \boldsymbol{z}'))\\ &\cdot l_{t'\boldsymbol{q}'}\,\frac{\delta}{\delta l_{t\boldsymbol{q}}}\,\frac{\delta}{\delta b_{s\boldsymbol{k}}}\,\mathrm{d}r\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}y'\,\mathrm{d}z'\,\mathrm{d}k\,\mathrm{d}q\,\mathrm{d}q'\end{aligned} \tag{8.8}$$

where the full notation is used. In particular (t) and (t') denote the various spin directions of the composite fermions while (s) denotes the class index of the boson. Furthermore for brevity we write $\mathrm{d}q \equiv \mathrm{d}^3 q$ etc. here and in the following and define $\hat V := G^0\,V^{\mathrm{a}}$ in accordance with (1.13).

Observing the special form of the cluster wave functions (6.14), (6.17), (6.18) and (7.12), (7.15), (7.16) after integration over $\boldsymbol{r}, \boldsymbol{x}, \boldsymbol{y}$ we obtain

$$\begin{aligned}(\mathscr{H}_{bl}^{1})_{\mathrm{a}}^{\mathrm{I}} = &\sum_{stt'} K(s, t, t')\,m^{15/2}\int e^{i\boldsymbol{k}\boldsymbol{z}}\,e^{i\boldsymbol{q}(\boldsymbol{z}+\boldsymbol{y}'+\boldsymbol{z}')1/3}\\ &\cdot e^{-\frac{1}{2}m^2(y'-z')^2}\,e^{-\frac{1}{2}m^2[z-\frac{1}{2}(y'+z')]^2}\,e^{-i\boldsymbol{q}'(\boldsymbol{z}+\boldsymbol{y}'+\boldsymbol{z}')1/3}\\ &\cdot e^{-\frac{1}{2}m^2(y'-z')^2}\,e^{-1/2\,m^2[z-\frac{1}{2}(y'+z')]^2}\\ &\cdot l_{t'\boldsymbol{q}'}\,\frac{\delta}{\delta l_{t\boldsymbol{q}}}\,\frac{\delta}{\delta b_{s\boldsymbol{k}}}\,\mathrm{d}z\,\mathrm{d}y'\,\mathrm{d}z'\,\mathrm{d}k\,\mathrm{d}q\,\mathrm{d}q'\end{aligned} \tag{8.9}$$

with

$$K(s, t, t') := \sum_{\substack{z r r' r''\\ u v}} \hat V_{\alpha\beta\gamma\delta}(-1)^z\,U^{r''r'}\,\chi_{\delta\gamma} \cdot U^{ruv}\,\chi^{t}_{\beta\varkappa\varkappa'}\,S^{zuv}\,\bar\chi^{t'}_{\alpha\varkappa\varkappa'}\,. \tag{8.10}$$

Direct calculation of (8.9) yields

$$(\mathscr{H}_{bl}^1)_a^1 = \sum_{stt'} K(s,t,t')\, m^{3/2} \int \delta(\boldsymbol{k}+\boldsymbol{q}-\boldsymbol{q}') \cdot l_{t'\boldsymbol{q}'} \frac{\delta}{\delta l_{t\boldsymbol{q}}} \frac{\delta}{\delta b_{s\boldsymbol{k}}} \mathrm{d}k\, \mathrm{d}q\, \mathrm{d}q'. \quad (8.11)$$

Since the multicenter integrals in (8.9) cannot exactly be solved, beyond the leading term (8.11) there appear terms of the same structure but multiplied by negative powers of m. These terms describe the deviation from the pointlike structure of the particles, i.e. are related to their finite spatial (and temporal) extension. Due to the smallness of m^{-n} these therms are very small compared with (8.11) and therefore are not explicitly given here.

The algebraic contributions contained in (8.10) can be exactly calculated. From (6.15) it follows

$$\sum_{r''r'} U^{r''r'} = 2(\tfrac{3}{2})^{-1/2}\, \eta^{-1} \quad (8.12)$$

and from (7.13) and (7.14)

$$\sum_{zrur} (-1)^z\, U^{rur}\, S^{zur} = -(16 + \tfrac{32}{9})\, \eta^{-1}. \quad (8.13)$$

We calculate the remaining spin part of (8.10) by means of $\hat{V}_{\alpha\beta\gamma\delta}$ which follows directly by comparison with (1.13). For a scalar interaction it is

$$\hat{V}_{\alpha\beta\gamma\delta} = \gamma^0_{\alpha\beta}\,\delta_{\gamma\delta} - \gamma^0_{\alpha\delta}\,\delta_{\gamma\beta} - \gamma^0_{\gamma\beta}\,\delta_{\alpha\delta} + \gamma^0_{\gamma\delta}\,\delta_{\alpha\beta}. \quad (8.14)$$

With the definition

$$\Lambda^{tt'}_{\alpha\beta} := \chi^{t'}_{\beta\varkappa\varkappa'}(\boldsymbol{q}')\, \bar{\chi}^{t}_{\alpha\varkappa\varkappa'}(\boldsymbol{q}) \quad (8.15)$$

we then obtain

$$\hat{V}_{\alpha\beta\gamma\delta}\, \chi_{\delta\gamma}\, \chi^{t'}_{\beta\varkappa\varkappa'}\, \bar{\chi}^{t}_{\alpha\varkappa\varkappa'} = \mathrm{tr}(\gamma^0\Lambda)\,\mathrm{tr}(\chi) - \mathrm{tr}[\Lambda(\gamma^0\chi + \chi\gamma^0)] + \mathrm{tr}(\Lambda)\,\mathrm{tr}(\gamma^0\chi). \quad (8.16)$$

This equation need be evaluated only in the rest frame of the boson, since any process of the kind (8.11) can be transformed into this rest frame. In this frame, i.e. for $\boldsymbol{k} \equiv 0$, the relation $\chi_{\alpha\beta} = -\chi_{\beta\alpha}$ is exactly valid independent of our approximation of the boson wave functions. If only scalar or pseudoscalar bosons are admitted the only antisymmetric spintensor is given by $\chi \equiv \gamma^5\gamma^0$, i.e. a pseudoscalar boson. Physically this tensor describes the spin-pairing of opposite spins of a particle-antiparticle pair at rest, i.e.

$$\chi = (\gamma^5\gamma^0)_{\alpha\beta} \equiv [u^+_\alpha(0)\, v^-_\beta(0) + u^-_\alpha(0)\, v^+_\beta(0)]_{\mathrm{as}}. \quad (8.17)$$

With this choice it can easily be seen that (8.16) vanishes. But if a process vanishes in the rest frame it vanishes everywhere. Thus we have $K \equiv 0$ and therefore

$$(\mathscr{H}_{bl}^1)_a^1 \equiv 0, \quad (8.18)$$

a result which is independent of any boson approximation.

The second term of (5.15) (denoted with index b) is defined by

$$(\mathscr{H}_{bl}^1)_b^1 := \left(\sum_{nqq'} \hat{V}_{\alpha\beta\gamma\delta}\, 6\, C_q^{\beta s u}\, C_n^{v\gamma}\, R^{q'}_{\alpha u v}\, l_{q'} \frac{\delta}{\delta l_q} \frac{\delta}{\delta b_n} \right)^1. \quad (8.19)$$

This term contains the algebraic part

$$K(s,t,t') := \sum_{\substack{zrr'r''\\ uv}} \hat{V}_{\alpha\beta\gamma\delta} (-1)^z\, U^{rr'}\, \chi_{\varkappa'\gamma} \cdot U^{rr''u}\, \chi^t_{\beta\delta\varkappa}\, S^{zuv}\, \bar{\chi}^t_{\alpha\varkappa\varkappa'} \quad (8.20)$$

and with (6.15), (7.13) and (7.14) direct calculation yields

$$\sum_{zrr'r''} (-1)^z\, U^{rr'}\, U^{rr''u}\, S^{zuv} = -\eta^{-2}[8(3+\tfrac{4}{3}\sqrt{2}) + \tfrac{8}{3}(\tfrac{8}{3} + 4\sqrt{2})] - 4\eta^{-4}. \quad (8.21)$$

In contrast to the first term the spin sum does in general not vanish. However, as the results of such calculations depend on the explicit form of the wave functions we do not attempt to specify them in more detail. Rather we assume the spin part to be unequal zero and try to make this plausible by using a simplified model. We assume the spin part of the three-fermion cluster to result from the pairing of antiparallel spins of a particle-antiparticle system combined with a single fermion spin, i.e. from the fusion of a boson with an elementary fermion. In this simplified case we have

$$\chi^t_{\alpha\beta\gamma}(\boldsymbol{q}) = \psi^t_\alpha(\boldsymbol{q})\, \hat{\chi}_{\beta\gamma}(\boldsymbol{q}) \quad (8.21)$$

and in the rest frame of the boson $b_{\boldsymbol{k}}$ with $\boldsymbol{k} \equiv 0$ by means of (8.14) the spin sum gives

$$\hat{V}_{\alpha\beta\gamma\delta}\, \chi_{\varkappa'\gamma}\, \chi^{t'}_{\beta\delta\varkappa}(\boldsymbol{q})\, \bar{\chi}^t_{\alpha\varkappa\varkappa'}(\boldsymbol{q}) = (\psi^t)^+ \cdot \psi^{t'}\, \mathrm{tr}(\hat{\chi}\hat{\chi}\chi) - \bar{\psi}^t[\gamma^0(\hat{\chi}\hat{\chi}\chi)]_+\, \psi^{t'} + \bar{\psi}^{t'} \cdot \psi^{t'}\, \mathrm{tr}(\gamma^0\hat{\chi}\hat{\chi}\chi) \neq 0. \quad (8.22)$$

If the space coordinate integrations etc. are performed in a similar way to the preceding case, we eventually get the formula

$$(\mathscr{H}_{bl}^1)_a^1 = \sum_{stt'} K(s,t,t')_{\mathrm{spin}}\, m^{3/2}\, \eta^{-2} \cdot \int \delta(\boldsymbol{k}+\boldsymbol{q}-\boldsymbol{q}')\, l_{t'\boldsymbol{q}'} \frac{\delta}{\delta l_{t\boldsymbol{q}}} \frac{\delta}{\delta b_{s\boldsymbol{k}}} \mathrm{d}k\, \mathrm{d}q\, \mathrm{d}q', \quad (8.23)$$

where it has to be observed that $K(s,t,t')_{\text{spin}}$ does not depend on s, i.e. $K(s,t,t')_{\text{spin}} \equiv K(t,t')_{\text{spin}}$.

In the following we give only the final expressions of our calculations. In the course of these calculations the sums over the auxiliary fields were performed exactly and the integrations were done along similar lines as demonstrated in the preceding example, while the spin sums in general were not evaluated at all. For brevity we suppress the spin state indices etc., i.e. we write $K \equiv K(s,t,t')$ etc.

We then obtain

$$(\mathscr{H}^2_{b\bar l})^{\mathrm{I}} = K^2_{\bar 3}\, m^{3/2}\, \eta^{-2} \int \delta(\boldsymbol{q}-\boldsymbol{k}-\boldsymbol{q}')\, b_k\, l_{q'} \frac{\delta}{\delta l_q}\, \mathrm{d}k\, \mathrm{d}q\, \mathrm{d}q' \tag{8.24}$$

with $K^2_{\bar 3} \equiv K(s,t,t')_{\text{spin}} \equiv 0$, i.e.

$$(\mathscr{H}^2_{b\bar l})^{\mathrm{I}} \equiv 0, \tag{8.25}$$

$$(\mathscr{H}^3_{b\bar l})^{\mathrm{I}} = K^3_{\bar 3}\, \eta^{-2} \int \delta(\boldsymbol{k}-\boldsymbol{k}'+\boldsymbol{q}-\boldsymbol{q}') \cdot b_{k'}\, l_{q'} \frac{\delta}{\delta l_q} \frac{\delta}{\delta b_k}\, \mathrm{d}q\, \mathrm{d}q' \mathrm{d}k\, \mathrm{d}k', \tag{8.26}$$

$$(\mathscr{H}^4_{b\bar l})^{\mathrm{I}} = K^4_{\bar 3}\, m^{-3/2}\, \eta^{-2} \int \delta(\boldsymbol{q}-\boldsymbol{q}'+\boldsymbol{k}-\boldsymbol{k}'-\boldsymbol{k}'') \cdot l_{q'}\, b_{k''} \frac{\delta}{\delta b_{k'}} \frac{\delta}{\delta b_k} \frac{\delta}{\delta l_q}\, \mathrm{d}k\, \mathrm{d}k'\, \mathrm{d}k''\, \mathrm{d}q\, \mathrm{d}q' \tag{8.27}$$

$$(\mathscr{H}^5_{b\bar l})^{\mathrm{I}} = K^5_{\bar 3}\, m^{-3/2}\, \eta^{-2} \int \delta(\boldsymbol{q}+\boldsymbol{q}'+\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{p}') \cdot l_p\, l_{p'} \frac{\delta}{\delta l_{q'}} \frac{\delta}{\delta l_q} \frac{\delta}{\delta b_k}\, \mathrm{d}q\, \mathrm{d}q'\, \mathrm{d}p\, \mathrm{d}p'\, \mathrm{d}k, \tag{8.28}$$

$$(\mathscr{H}^2_{b l\bar l})^{\mathrm{I}} = K^2_{\bar 4}\, \eta^{-2} \int \delta(\boldsymbol{q}-\boldsymbol{w}-\boldsymbol{k}-\boldsymbol{k}') \cdot b_k\, b_{k'} \frac{\delta}{\delta \bar l_w} \frac{\delta}{\delta l_q}\, \mathrm{d}w\, \mathrm{d}q\, \mathrm{d}k\, \mathrm{d}k', \tag{8.29}$$

$$(\mathscr{H}^3_{b l\bar l})^{\mathrm{I}} = K^3_{\bar 4}\, m^{-3/2}\, \eta^{-2} \int \delta(\boldsymbol{k}+\boldsymbol{w}-\boldsymbol{w}'+\boldsymbol{q}-\boldsymbol{q}') \cdot l_q\, \bar l_{w'} \frac{\delta}{\delta l_q} \frac{\delta}{\delta \bar l_w} \frac{\delta}{\delta b_k}\, \mathrm{d}q\, \mathrm{d}q'\, \mathrm{d}w\, \mathrm{d}w'\, \mathrm{d}k, \tag{8.30}$$

$$(\mathscr{H}^4_{b l\bar l})^{\mathrm{I}} = K^4_{\bar 4}\, m^{-9}\, \eta^{-2} \int \delta(\boldsymbol{q}+\boldsymbol{q}'-\boldsymbol{q}''+\boldsymbol{w}-\boldsymbol{k}-\boldsymbol{k}'') \cdot l_{q''}\, b_k\, b_{k'} \frac{\delta}{\delta l_{q'}} \frac{\delta}{\delta \bar l_w} \frac{\delta}{\delta l_q}\, \mathrm{d}q\, \mathrm{d}q'\, \mathrm{d}q''\, \mathrm{d}w\, \mathrm{d}k\, \mathrm{d}k', \tag{8.31}$$

$$(\mathscr{H}^1_{bb})^{\mathrm{I}} = K^1_{\bar 1}\, \eta^{-2} \int \delta(\boldsymbol{k}+\boldsymbol{k}'-\boldsymbol{q}-\boldsymbol{q}') \cdot b_k\, b_{k'} \frac{\delta}{\delta b_q} \frac{\delta}{\delta b_{q'}}\, \mathrm{d}k\, \mathrm{d}k'\, \mathrm{d}q\, \mathrm{d}q', \tag{8.32}$$

$$(\mathscr{H}^2_{b\bar b})^{\mathrm{I}} = K^2_{\bar 1}\, m^{-3/2}\, \eta^{-2} \int \delta(\boldsymbol{k}+\boldsymbol{k}'+\boldsymbol{k}''-\boldsymbol{q}-\boldsymbol{q}') \cdot b_q\, b_{q'} \frac{\delta}{\delta b_{k''}} \frac{\delta}{\delta b_{k'}} \frac{\delta}{\delta b_k}\, \mathrm{d}k\, \mathrm{d}k'\, \mathrm{d}k''\, \mathrm{d}q\, \mathrm{d}q', \tag{8.33}$$

$$(\mathscr{H}^3_{b\bar b})^{\mathrm{I}} = K^3_{\bar 1}\, m^{3/2}\, \eta^{-2} \int \delta(\boldsymbol{k}+\boldsymbol{k}'-\boldsymbol{q}) \cdot b_q \frac{\delta}{\delta b_{k'}} \frac{\delta}{\delta b_k}\, \mathrm{d}k\, \mathrm{d}k'\, \mathrm{d}q.$$

This process is energetically forbidden, hence

$$(\mathscr{H}^3_{b\bar b})^{\mathrm{I}} \equiv 0, \tag{8.34}$$

$$(\mathscr{H}^4_{b\bar b})^{\mathrm{I}} = K^4_{\bar 1}\, m^{3/2}\, \eta^{-2} \int \delta(\boldsymbol{k}-\boldsymbol{q}-\boldsymbol{q}') \cdot b_q\, b_{q'} \frac{\delta}{\delta b_k}\, \mathrm{d}k\, \mathrm{d}q\, \mathrm{d}q'. \tag{8.35}$$

This process is energetically forbidden, hence

$$(\mathscr{H}^4_{b\bar b})^{\mathrm{I}} \equiv 0. \tag{8.36}$$

Due to the vanishing of the spin part it is furthermore

$$(\mathscr{H}^5_{b\bar b})^{\mathrm{I}} \equiv 0 \tag{8.37}$$

while for the fermion-fermion interaction the following expression results

$$(\mathscr{H}^1_{l l})^{\mathrm{I}} = K^1_{\bar 2}\, \eta^{-2} \int \delta(\boldsymbol{q}+\boldsymbol{q}'-\boldsymbol{p}-\boldsymbol{p}') \cdot l_p\, l_{p'} \frac{\delta}{\delta l_{q'}} \frac{\delta}{\delta l_q}\, \mathrm{d}p\, \mathrm{d}p'\, \mathrm{d}q\, \mathrm{d}q'. \tag{8.38}$$

The terms for $b\bar l$ etc. processes can be obtained from the terms given here by simply changing l into $\bar l$ etc. and shall not explicitly be written out here.

Finally, we treat the expression $\mathscr{H}_{l\bar l}$ of (5.23) which is more difficult to evaluate than the preceding terms. It is given by the formula

$$(\mathscr{H}_{l\bar l})^{\mathrm{I}} = \Big(-\sum_{\substack{qq'\\ ww'}} V_{\alpha\beta\gamma\delta}\, F_{\beta\beta'}\, 3\, C^{\delta s t}_q\, 3\, \bar C^{\gamma u v}_w\, R^{q'}_{\alpha s t}\, \bar R^{w'}_{\beta' u v}\, l_{q'}\, \bar l_{w'} \frac{\delta}{\delta \bar l_w} \frac{\delta}{\delta l_q} \Big)^{\mathrm{I}} \tag{8.39}$$

$$\equiv \sum_{\substack{z r r' r''\\ u v\\ u' v'}} \sum_{\substack{s s'\\ t t'}} (-1)^z \int \hat V_{\alpha\beta\gamma\delta}\, \delta(\boldsymbol{r}-\boldsymbol{x})\, \delta(\boldsymbol{r}-\boldsymbol{y})\, \delta(\boldsymbol{r}-\boldsymbol{z})\, F^{r''}_{\delta\delta'}(\boldsymbol{z}-\boldsymbol{z}')\, 3\, e^{i\boldsymbol{q}(\boldsymbol{x}+\boldsymbol{x}'+\boldsymbol{x}'')1/3} \varphi_{0,t}(\boldsymbol{x}' \underset{\beta}{\overset{r}{-}} \boldsymbol{x}'', \boldsymbol{x} \underset{\lambda}{\overset{u}{-}} \tfrac{1}{2}(\boldsymbol{x}' \underset{\lambda'}{\overset{v}{+}} \boldsymbol{x}''))$$

$$\cdot\, e^{-i\boldsymbol{k}(\boldsymbol{y}+\boldsymbol{y}'+\boldsymbol{y}'')1/3} \bar\varphi_{0,s}(\boldsymbol{y}' \underset{\gamma}{\overset{r'}{-}} \boldsymbol{y}'', \boldsymbol{y} \underset{\varkappa}{\overset{u'}{-}} \tfrac{1}{2}(\boldsymbol{y}' \underset{\varkappa'}{\overset{v'}{+}} \boldsymbol{y}''))\, 3\, e^{-i\boldsymbol{q}'(\boldsymbol{r}+\boldsymbol{x}'+\boldsymbol{x}'')1/3} \bar\varphi_{0,t'}(\boldsymbol{x}' \underset{\alpha}{\overset{z}{-}} \boldsymbol{x}'', \boldsymbol{r} \underset{\lambda}{\overset{u}{-}} \tfrac{1}{2}(\boldsymbol{x}' \underset{\lambda'}{\overset{v}{+}} \boldsymbol{x}''))\, e^{i\boldsymbol{k}'(\boldsymbol{z}'+\boldsymbol{y}'+\boldsymbol{y}'')1/3}$$

$$\cdot\, \varphi_{0,s}(\boldsymbol{y}' \underset{\delta'}{\overset{r''}{-}} \boldsymbol{y}'', \boldsymbol{z}' \underset{\varkappa}{\overset{u'}{-}} \tfrac{1}{2}(\boldsymbol{y}' \underset{\varkappa'}{\overset{v'}{+}} \boldsymbol{y}''))\, l_{t'\boldsymbol{q}'}\, \bar l_{s'\boldsymbol{k}'} \frac{\delta}{\delta \bar l_{s\boldsymbol{k}}} \frac{\delta}{\delta l_{t\boldsymbol{q}}}\, \mathrm{d}r\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z\, \mathrm{d}z'\, \mathrm{d}x'\, \mathrm{d}y'\, \mathrm{d}x''\mathrm{d}y''\, \mathrm{d}q\, \mathrm{d}q'\, \mathrm{d}k\, \mathrm{d}k'.$$

Observing the special form of the cluster wave functions (7.12), (7.15), (7.16) we can treat the expression

$$K := \sum_{\substack{zrr'r'' \\ uvu'v'}} (-1)^z F^{r''} U^{ruv} S^{zuv} U^{r'u'v'} S^{r''u'v'} \tag{8.40}$$

separately. By direct calculation we obtain

$$\sum_{rzuv} (-1)^z U^{ruv} S^{zuv} = -\left(16 + \frac{32}{9}\right) \eta^{-1} \tag{8.41}$$

and

$$\sum_{\substack{r''r' \\ uv}} F^{r''} U^{r'uv} S^{r''uv} = -\left(36 + \frac{12}{3}\right)(F^1 + F^2) + \left(8 + \frac{16}{9}\right)(F^1 - F^2)\,\eta^{-1}. \tag{8.42}$$

If for the latter expression the approximations (8.6) and (8.7) are taken into account, apart from numerical factors (8.42) yields

$$\sum_{\substack{r''r' \\ uv}} F^{r''}_{\delta\delta'}(z - z')\, U^{r'uv} S^{r''uv} \approx \eta^{-1}\, \delta(z - z')\, \delta_{\delta\delta'}. \tag{8.43}$$

For further evaluation we use the notation $l_t(\boldsymbol{q}) := l_{t\boldsymbol{q}}$ etc. and introduce the definition

$$l_\varrho(\boldsymbol{q}) := \sum_t \psi^t_\varrho(\boldsymbol{q})\, l_t(\boldsymbol{q}), \tag{8.44}$$

where $\{\psi^t_\varrho(\boldsymbol{q})\}$ is a complete set of Dirac spinors for the center of mass motion of the composite fermion with center of mass momentum $\boldsymbol{q}$. The inversion of (8.44) yields

$$l_t(\boldsymbol{q}) = \sum_\varrho \bar\psi^t_\varrho(\boldsymbol{q})\, l_\varrho(\boldsymbol{q}) \tag{8.45}$$

if the metric tensor of the spin space is incorporated in $\bar\psi^t$.

Since the total wave function $\varphi_{1,0}(\underset{\alpha}{\boldsymbol{r}}, \underset{\beta}{\boldsymbol{r}'}, \underset{\gamma}{\boldsymbol{r}''} | t\boldsymbol{q})$ must be antisymmetric, according to (7.15) and (7.16) this antisymmetry must be provided by the spin part $\chi^t_{\alpha\beta\gamma}$. As usual we choose a suitable spin configuration and antisymmetrize afterwards. This will be expressed by the notation $\{\chi^t_{\alpha\beta\gamma}\}_{\text{as}}$. The spin part can be expanded in terms of the γ-algebra. This gives for the original configuration

$$\chi^t_{\alpha\beta\gamma} \equiv \chi^t_{\alpha\beta\gamma}(\boldsymbol{q}) = \sum_{l=1}^{15} a^t_\alpha(\boldsymbol{q}, l)\, \Gamma^l_{\beta\gamma}. \tag{8.46}$$

If we assume that the composite fermion is a bound state of a scalar boson and a spin 1/2 fermion then only Γ^1 occurs in the expansion (8.46) and observing $a^t_\alpha(\boldsymbol{q}, 1) \equiv \psi^t_\alpha(\boldsymbol{q})$ we obtain

$$\sum_t \chi^t_{\alpha\beta\gamma}\, l_t(\boldsymbol{q}) = \sum_t a^t_\alpha(\boldsymbol{q}, 1)\, \bar\psi^t_\varrho(\boldsymbol{q})\, \Gamma^1_{\beta\gamma}\, l_\varrho(\boldsymbol{q}) = \Gamma^1_{\alpha\varrho} \Gamma^1_{\beta\gamma}\, l_\varrho(\boldsymbol{q}). \tag{8.47}$$

With these expressions we can rearrange the spin part of (8.39) in the following way

$$\sum_{\substack{ss' \\ tt'}} \hat V_{\alpha\beta\gamma\delta}\, \delta_{\delta\delta'} \{\chi^s_{\beta\lambda\lambda'}\}_{\text{as}} \{\bar\chi^{s'}_{\gamma\varkappa\varkappa'}\}_{\text{as}} \{\chi^t_{\alpha\lambda\lambda'}\}_{\text{as}} \{\bar\chi^{t'}_{\delta'\varkappa\varkappa'}\}_{\text{as}}\, l_t(\boldsymbol{q}')\, \bar l_{t'}(\boldsymbol{k}') \frac{\delta}{\delta l_s(\boldsymbol{q})} \frac{\delta}{\delta \bar l_{s'}(\boldsymbol{k})} = \sum_{ss'} K_{\varrho\varrho'}(s, s')\, l_\varrho(\boldsymbol{q}')\, \bar l_{\varrho'}(\boldsymbol{k}') \frac{\delta}{\delta l_s(\boldsymbol{q})} \frac{\delta}{\delta \bar l_{s'}(\boldsymbol{k})} \tag{8.48}$$

with

$$K_{\varrho\varrho'}(s, s') := \hat V_{\alpha\beta\gamma\delta} \{\chi^s_{\beta\lambda\lambda'}\}_{\text{as}} \{\bar\chi^{s'}_{\gamma\varkappa\varkappa'}\}_{\text{as}} \cdot \{\Gamma^1_{\alpha\varrho} \Gamma^1_{\lambda\lambda'}\}_{\text{as}} \{\Gamma^1_{\delta\varrho'} \Gamma^1_{\varkappa\varkappa'}\}_{\text{as}}, \tag{8.49}$$

where ϱ and ϱ' are excluded from antisymmetrization. If we define by $l_\varrho(\boldsymbol{r}) = \mathscr{F}[l_\varrho(\boldsymbol{q})]$ etc. the Fourier transform of (8.44) then after integration over $\boldsymbol{r}, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$ and with (8.40)–(8.49) the expression (8.39) goes over into

$$\begin{aligned}
(\mathscr{H}_{l\bar l})^1 = \sum_{ss'} \eta^{-2} \int & K_{\varrho\varrho'}(s, s')\, \chi_0(\boldsymbol{x}' - \boldsymbol{x}'', \boldsymbol{z} - \tfrac{1}{2}(\boldsymbol{x}' + \boldsymbol{x}''))\, \chi_0(\boldsymbol{y}' - \boldsymbol{y}'', \boldsymbol{z} - \tfrac{1}{2}(\boldsymbol{y}' + \boldsymbol{y}''))\, e^{-i\boldsymbol{q}'(\boldsymbol{z}+\boldsymbol{x}'+\boldsymbol{x}'')1/3} \\
&\cdot \chi_0(\boldsymbol{x}' - \boldsymbol{x}'', \boldsymbol{z} - \tfrac{1}{2}(\boldsymbol{x}' + \boldsymbol{x}''))\, e^{i\boldsymbol{k}'(\boldsymbol{z}+\boldsymbol{y}'+\boldsymbol{y}'')1/3} \chi_0(\boldsymbol{y}' - \boldsymbol{y}'', \boldsymbol{z} - \tfrac{1}{2}(\boldsymbol{y}' + \boldsymbol{y}''))\, l_\varrho[(\boldsymbol{z} + \boldsymbol{x}' + \boldsymbol{x}'')\,1/3] \\
&\cdot \bar l_{\varrho'}[(\boldsymbol{z} + \boldsymbol{y}' + \boldsymbol{y}'')\,1/3] \frac{\delta}{\delta l_{s\boldsymbol{q}}} \frac{\delta}{\delta \bar l_{s'\boldsymbol{k}}}\, \mathrm{d}z\, \mathrm{d}x'\, \mathrm{d}x''\, \mathrm{d}y'\, \mathrm{d}y''\, \mathrm{d}q\, \mathrm{d}k.
\end{aligned} \tag{8.50}$$

For the final step of the evaluation of (8.50) we need the following lemma:

Lemma: The product of the fermion and antifermion cluster operators $l_\varrho(\boldsymbol{r})$ and $l_{\varrho'}(\boldsymbol{r}')$ respectively allows the representation

$$l_\varrho(\boldsymbol{r})\, \bar l_{\varrho'}(\boldsymbol{r}') = \sum_s \int e^{i\boldsymbol{k}(\boldsymbol{r}+\boldsymbol{r}')1/2} \chi_{0,s}(\boldsymbol{r} - \boldsymbol{r}')_{\varrho\varrho'} (-1)^s\, b_{s\boldsymbol{k}}\, \mathrm{d}k \tag{8.51}$$

in terms of boson cluster operators.

Proof: Owing to the Theorem 1 the set of one-time boson wave functions $\{C^{uv}_n\}$ can uniquely be continued to the set $\{B^{uv}_n\}$ of many-time boson wave functions. The latter set can be used to define the manifest covariant boson cluster operators

$$\hat{b}_n := B_n^{uv} j_u j_v \tag{8.52}$$

with u, v four-dimensional coordinates etc. The states

$$|\mathfrak{F}(j, n_1 \dots n_N)\rangle := (N!)^{-1/2} \hat{b}_{n_1}(j) \dots \hat{b}_{n_N}(j) |0\rangle \tag{8.53}$$

are N-boson functional states which satisfy the subsidiary conditions (3.3), i.e.

$$\mathscr{P}_\mu |\mathfrak{F}(j, n_1 \dots n_N)\rangle = \sum_{i=1}^{n} p_\mu^i |\mathfrak{F}(j, n_1 \dots n_N)\rangle \tag{8.54}$$

but which do not satisfy the dynamical equation (3.2). Hence the set of many boson states $\{|\mathfrak{F}(j, n_1 \dots n_N)\rangle \,\forall N\}$ is only characterized by the relations (8.54) but not by being solutions of (3.2).

For the derivation of (8.51) we need equivalent boson state representations. Obviously this equivalence can only be defined with respect to the subsidiary conditions (8.54) as no other equivalence criteria are available (apart from the spin subsidiary conditions which are omitted for brevity). For instance the time-ordered functionals

$$|\mathfrak{T}(j, n_1 \dots n_N)\rangle = \exp(-jFj)\, |\mathfrak{F}(j, n_1 \dots n_N)\rangle \tag{8.55}$$

satisfy (8.54) as well. If the dynamical equation, however, plays no role, then it follows immediately that even by the single terms of the power series expansion of (8.55) transformed boson cluster operators

$$\hat{b}_n^{(v)} := (-jFj)^v\, \hat{b}_n\,; \quad v = 1, \dots \tag{8.56}$$

can be defined such that the states

$$|\mathfrak{F}(j, n_1 \dots n_N)\rangle^{(v)} := (N!)^{-1/2} \hat{b}_{n_1}^{(v)}(j) \dots \hat{b}_{n_N}^{(v)}(j) |0\rangle \tag{8.57}$$

satisfy (8.54), too. Thus the set of states (8.57) and boson cluster operators (8.56) can be equally well used for the description of many boson systems and, without changing the content of the theory, in a cluster representation $\hat{b}_n^{(v)}$ can be replaced by $\hat{b}_n$. In the one-time limit we therefore obtain for $v = 2$

$$\begin{aligned} \operatorname{Lim} \hat{b}_n^{(2)} &= b_n^{(2)} \\ &= C_n^{\alpha\beta} F^{\alpha'\beta'} F^{\alpha''\beta''} j_\alpha j_{\alpha'} j_{\alpha''} \bar{j}_\beta \bar{j}_{\beta'} \bar{j}_{\beta''} \mathrel{\hat{=}} b_n . \end{aligned} \tag{8.58}$$

By means of (3.15a) and (3.15b) this can be rewritten as

$$b_n \mathrel{\hat{=}} \sum_{qq'} C_n^{\alpha\beta} F^{\alpha'\beta'} F^{\alpha''\beta''} R^q_{\alpha\alpha'\alpha''} \bar{R}^{q'}_{\beta\beta'\beta''} l_q \bar{l}_{q'} . \tag{8.59}$$

If this formula is written in the full notation the auxiliary field algebra can be separated and directly calculated. It yields

$$\begin{aligned} K = & \sum_{\substack{u v \\ u' v' u'' v''}} U^{uv} F^{u'v'} F^{u''v''} \delta_{u'v'} \delta_{u''v''} S^{uu'u''} S^{vv'v''} \\ = & (F^1 + F^2)^2 \left(\frac{81}{4} + \frac{1}{\sqrt{2}} \frac{9}{2} + \frac{1}{8} \right) \eta^2 \\ & + 2(F^1 - F^2)(F^1 + F^2) \left(\frac{1}{4} + \frac{1}{\sqrt{2}} \frac{1}{3} + \frac{1}{18} \right) \\ & + (F^1 - F^2)^2 \left(\frac{1}{9} + \frac{1}{\sqrt{2}} \frac{1}{3} + \frac{1}{2} \right) \eta^{-2} \end{aligned} \tag{8.60}$$

and with (8.6) and (8.7) we obtain for (8.59)

$$\begin{aligned} b_{sk} \mathrel{\hat{=}} & \int e^{-ik(x+y)1/2} \chi_{0,s}(\underset{\alpha}{x} - \underset{\beta}{y})\, e^{-iq(x+x'+x'')1/3} \\ & \cdot \chi_0(x' - x'', x - \tfrac{1}{2}(x' + x'')) \\ & \cdot e^{-iq'(y+x'+x'')1/3} \chi_0(x' - x'', y - \tfrac{1}{2}(x' + x'')) \\ & \cdot \sum_{tt'} \chi^t_{\alpha\alpha'\alpha''} \bar{\chi}^{t'}_{\beta\alpha'\alpha''} l_t(q)\, \bar{l}_{t'}(q')\, \mathrm{d}x\, \mathrm{d}x' \mathrm{d}y\, \mathrm{d}y'\, \mathrm{d}q\, \mathrm{d}q' . \end{aligned} \tag{8.61}$$

By similar considerations as above we have

$$\sum_{tt'} \chi^t_{\alpha\alpha'\alpha''} \bar{\chi}^{t'}_{\beta\alpha'\alpha''} l_t(q)\, \bar{l}_{t'}(q) = l_\alpha(q)\, \bar{l}_\beta(q') \tag{8.62}$$

and direct calculation of (8.61) leads to

$$b_{sk} \mathrel{\hat{=}} \int e^{-ik(x+y)1/2} \chi_{0,s}(\underset{\alpha}{x} - \underset{\beta}{y})\, l_\alpha(x)\, \bar{l}_\beta(y)\, \mathrm{d}x\, \mathrm{d}y . \tag{8.63}$$

If it is observed that for inversion the class index is negative for the negative class, we obtain from (8.63) formula (8.51). Q.E.D.

As we work in the cluster representation, in the following we can replace $\mathrel{\hat{=}}$ by $=$ in (8.63). Substitution of (8.51) into (8.50) yields after straightforward integrations

$$\begin{aligned} (\mathscr{H}_{l\bar{l}})^{\mathrm{I}} = & \sum_{stt'} K(t, t')\, m^{3/2} \eta^{-2} \\ & \cdot \int \delta(p + k - q)(-1)^s\, b_{sp} \frac{\delta}{\delta \bar{l}_{tk}} \frac{\delta}{\delta l_{t'q}}\, \mathrm{d}k\, \mathrm{d}p\, \mathrm{d}q \end{aligned} \tag{8.64}$$

with

$$K(t, t') := K_{\varrho\varrho'}(t, t')\, \chi_{\varrho\varrho'} \tag{8.65}$$

With the derivation of formula (8.64) the discussion of $\mathscr{H}^{\mathrm{I}}$ is complete. The discussion about the effect of the inclusion of $\mathscr{H}^{\mathrm{II}}$ is performed in the next section.

9. Leading Term Approximation

The cluster expansion of the operator $\mathcal{H}$ of the basic functional energy representation (1.11) or (1.15) respectively is characterized by the decompositions (5.1), (5.6) and (8.1). The evaluation of the various terms of these decompositions in particular with respect to the interaction terms of $\mathcal{H}^{\mathrm{I}}$ showed that, apart from the various cluster operator combinations, three numerical constants m, η and g play an essential role. Due to the large values of m and η and the freedom in fixing Δm these constants are well suited for defining a hierarchy of interaction terms. Summarizing the results of our preceding calculations we have the following magnitude of the various operators (cf. Def. (6.10))

$$\begin{aligned}
&(\mathcal{H}_{b\bar{l}}^{1})^{\mathrm{I}} \sim m^{3/2}\,\eta^{-2}\,g; &&(\mathcal{H}_{b\bar{b}}^{1})^{\mathrm{I}} \sim m^{0}\,\eta^{-2}\,g\\
&(\mathcal{H}_{b\bar{l}}^{2})^{\mathrm{I}} = 0; &&(\mathcal{H}_{b\bar{b}}^{2})^{\mathrm{I}} \sim m^{-3/2}\,\eta^{-2}\,g\\
&(\mathcal{H}_{b\bar{l}}^{3})^{\mathrm{I}} \sim m^{0}\,\eta^{-2} g; &&(\mathcal{H}_{b\bar{b}}^{3})^{\mathrm{I}} = 0\\
&(\mathcal{H}_{b\bar{l}}^{4})^{\mathrm{I}} \sim m^{-3/2}\,\eta^{-2}\,g; &&(\mathcal{H}_{b\bar{b}}^{4})^{\mathrm{I}} = 0\\
&(\mathcal{H}_{b\bar{l}}^{5})^{\mathrm{I}} \sim m^{-3/2}\,\eta^{-2}\,g; &&(\mathcal{H}_{b\bar{b}}^{5})^{\mathrm{I}} = 0\\
&(\mathcal{H}_{bl\bar{l}}^{1})^{\mathrm{I}} \equiv (\mathcal{H}_{l\bar{l}}^{1})^{\mathrm{I}} \sim m^{3/2}\,\eta^{-2}\,g &&\qquad\qquad (9.1)\\
&(\mathcal{H}_{bl\bar{l}}^{2})^{\mathrm{I}} \sim m^{0}\,\eta^{-2}\,g; &&(\mathcal{H}_{ll}^{1})^{\mathrm{I}} \sim m^{0}\,\eta^{-2}\,g\\
&(\mathcal{H}_{bl\bar{l}}^{3})^{\mathrm{I}} \sim m^{-3/2}\,\eta^{-2}\,g;\\
&(\mathcal{H}_{bl\bar{l}}^{4})^{\mathrm{I}} \sim m^{-9}\,\eta^{-2}\,g
\end{aligned}$$

where the complete vanishing of some elements is due to spin sums or energy non-conservation respectively. The leading terms in this hierarchy of interactions are proportional to $m^{3/2}\,\eta^{-2}\,g$ while all other terms are smaller in magnitudes of $m^{-3/2}$ compared to these leading terms. Due to the large m values we are thus justified to neglect these terms. In this leading term approximation we then get the following functional energy cluster representation

$$\begin{aligned}
\mathcal{H}\,|\mathfrak{F}\rangle =& \sum_s \int E_{sq}^{b}\, b_{sq} \frac{\delta}{\delta b_{sq}}\,\mathrm{d}q\,|\mathfrak{F}\rangle + \sum_t \int E_{tq}^{f}\, l_{tq} \frac{\delta}{\delta l_{tq}}\,\mathrm{d}q\,|\mathfrak{F}\rangle + \sum_t \int E_{tq}^{\bar{f}}\, \bar{l}_{tq} \frac{\delta}{\delta \bar{l}_{tq}}\,\mathrm{d}q\,|\mathfrak{F}\rangle\\
&+\sum_{tt'} K(t,t')\, m^{3/2}\eta^{-2} g \int \delta(\boldsymbol{k}+\boldsymbol{q}-\boldsymbol{q}')\left[\frac{\delta}{\delta b_{+\boldsymbol{k}}}+\frac{\delta}{\delta b_{-\boldsymbol{k}}}\right]\cdot\left[l_{tq}\frac{\delta}{\delta l_{t'q'}}+\bar{l}_{tq}\frac{\delta}{\delta \bar{l}_{t'q'}}\right]\mathrm{d}k\,\mathrm{d}q\,\mathrm{d}q'\,|\mathfrak{F}\rangle \qquad (9.2)\\
&+\sum_{tt'} K'(t,t')\, m^{3/2}\,\eta^{-2}\,g \int \delta(\boldsymbol{k}+\boldsymbol{q}-\boldsymbol{q}')\,[b_{+\boldsymbol{k}}-b_{-\boldsymbol{k}}]\,\frac{\delta}{\delta \bar{l}_{tq}}\,\frac{\delta}{\delta l_{t'q'}}\,\mathrm{d}k\,\mathrm{d}q\,\mathrm{d}q'\,|\mathfrak{F}\rangle + \mathcal{H}^{\mathrm{II}}\,|\mathfrak{F}\rangle.
\end{aligned}$$

By comparison with (2.6) we see that the cluster representation of (1.11) in the leading term approximation yields the functional energy representation of a phenomenological meson-fermion coupling theory apart from the fermion and boson quantization terms. The latter terms cannot be expected to appear in a cluster map, as by introducing clusters the local field operators are softened into cluster operators. Hence a perfect map can only be achieved between representations where the quantization terms are removed. Such a removal can be performed in the functional equations of the phenomenological fields by a transition to normal ordered functionals

$$|\mathfrak{T}(\eta,\bar{\eta},J)\rangle = \exp\left[-\,\eta F^{f}\bar{\eta} - J F^{b} J\right]\,|\mathfrak{F}(\eta,\bar{\eta},J)\rangle. \qquad (9.3)$$

In the elementary fermion field functionals the transformation

$$|\mathfrak{T}(j,\bar{j})\rangle = \exp\left[-j F_2 \bar{j} - j\bar{j} F_4 j\bar{j} - jjj F_6 \bar{j}\bar{j}\bar{j}\right]\,|\mathfrak{Z}(j,\bar{j})\rangle \qquad (9.4)$$

corresponds to this transition. The further study of this map is postponed to subsequent papers. We only sketch the inclusion of $\mathcal{H}^{\mathrm{II}}$ into these calculations in order to complete our investigation.

The following statement holds:

The terms of $\mathcal{H}^{\mathrm{II}}$ contribute to all cluster reactions with expressions proportional to m^{-2} provided that for the total energy $E < m$ holds.

To explain this statement we observe that to each cluster independently of being a scattering state or a bound state a kinetic energy term appears in $\mathcal{H}$ which contains the cluster energies E^{b}, E^{f}, $E^{\bar{f}}$ respectively. For cluster scattering states these energies are all above the threshold of elementary fermion production. This means that for all energies below the elementary fermion production threshold no direct reactions with cluster scattering states are admitted due to energy conservation; i.e. if it is assumed that the total energy E is smaller than this threshold there is no contribution of cluster scattering states with respect to direct reactions. Therefore, the occurrence of elementary fermion cluster scattering states is restricted to off-mass shell intermediate states.

Concerning the influence of such intermediate states on the dynamics we re-express all cluster terms in (5.1) which describe elementary fermion scattering by their original sources. Then (5.1) goes over into

$$\mathscr{H}[b, l, \bar{l}] = \hat{\mathscr{H}}[b, l, \bar{l}, j, \bar{j}] \tag{9.5}$$

i.e. $\hat{\mathscr{H}}$ describes a system of bound states with small masses and of elementary fermions with very large masses which are all in interaction. As the original field equation (1.1) is renormalizable we assume that also the theory belonging to (9.5) is renormalizable. For this kind of theories Kazama and York-Peng Yao [91] published a basic paper in which they demonstrated that the total Green function for a system of light particles and of very heavy particles can be separated into two parts. The first part gives the Green functions of the light particles alone, while the second part contains contributions of both kinds of particles multiplied with the factor m^{-2} where m is the large mass of the heavy particles. If we transfer this result to our model we can identify $\mathscr{H}^{\mathrm{I}}$ with that part of $\mathscr{H}$ which leads to the light particle Green functions, while $\mathscr{H}^{\mathrm{II}}$ leads to the remaining Green functions, i.e. gives only corrections of order m^{-2}.

Although this result indicates that the influence of $\mathscr{H}^{\mathrm{II}}$ below the threshold $E < m$ can be approximately neglected, it is not satisfactory insofar as the leading term approximation is based on the factor $m^{3/2}\eta^{-2} g \equiv m^{-1/2}(\Delta m)^2 g$. Therefore, the results of Kazama and York-Peng Yao need a further elaboration for our model. In principle this must be done by extending our method which we used for the evaluation of bound states to include the scattering states, too. However, for brevity we postpone this problem to forthcoming papers. In the literature the study of the separation of heavy particle effects from light particle dynamics and the derivation of effective field theories for the light particles was initiated by Appelquist and Carazzone [92] who proved a decoupling theorem. Further papers about this topic were published by Ovrut and Schnitzer [93], Weisberger [94], Lee and Pac [95], Manoukian [96], Chang, Das, Li, Xian and Zhou [97], Akhoury and Yao [98], McKeon [99], Barbieri, Ferrara and Nanopoulos [100] etc.

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